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SELF–TUNING PREDICTIVE CONTROL OF NONLINEAR SERVO–MOTOR

Vladimír Bobál – Petr Chalupa – Marek Kubalčík – Petr Dostál *

The paper is focused on a design of a self-tuning predictive model control (STMPC) algorithm and its application to a control of a laboratory servo motor. The model predictive control algorithm considers constraints of a manipulated variable. An ARX model is used in the identification part of the self-tuning controller and its parameters are recursively estimated using the recursive least squares method with the directional forgetting. The control algorithm is based on the Generalised Predictive Control (GPC) method and the optimization was realized by minimization of a quadratic and absolute values objective functions. A recursive control algorithm was designed for computation of individual predictions by incorporating a receding horizon principle. Proposed predictive controllers were verified by a real-time control of highly nonlinear laboratory model — Amira DR300.

K e y w o r d s: non-linear system, servo-system, CARIMA model, self-tuning control, predictive control, real-time control

1 INTRODUCTION

Predictive control is one of successful methodologies which draw much research interest and attention over recent decades. First predictive control algorithms have been applied as an effective tool for control of multidimensional industrial processes with constraints more than 25 years ago. Because of its computational complexity, predictive control has traditionally been used for slow processes only. However, with the advances made in the computer technology over the last decade computational speed is not a major limitation for many real-life applications.

Predictive control methods have evolved in many different variants and under several names (Model Predictive Control (MPC), Generalized Predictive Control (GPC) [1], Receding Horizon Control (RHC) [2, 3]). Initially, all variants of the predictive control were developed independently. Several papers that try to clarify the connections between the variants and to consolidate them were published in early 90s of the last century [4–6]. Surveys of the present-day predictive control methods can be found in [7–13].

Theoretical research in the area of predictive control has a great impact on the industrial world and there are many applications of predictive control in industry. Its development has been significantly influenced by industrial practice. At present, predictive control with a number of real industrial applications belongs among the most often implemented modern industrial process control approaches. Fairly actual and extensive surveys of industrial applications of predictive control are presented in [14–17].

One of the major advantages of predictive control is its ability to do on-line constraints handling in a systematic way. Almost all industrial applications hold constraints of input, output and state space variables. The predictive control strategy therefore eliminates drawbacks of the other optimal methods like Linear Quadratic (LQ) and Linear Quadratic Gaussian (LQG) methods, which operate on a finite horizon without capability to handle constraints. In practical control problems, actuators are obviously limited in their operational ranges. This is also the case of the laboratory servo-motor Amira DR300 [18].

The aim of this contribution is an implementation of the self-tuning predictive controller including constraints of the manipulated variable for control of the objective laboratory equipment. An input-output CARIMA (Controlled Auto-Regressive Integrated Moving Average) model was chosen as a model describing the controlled process. Its parameters were estimated using the recursive least squares method with the directional forgetting. [19–21]. A quadratic cost function and absolute values cost function were used in the optimization part of the algorithm. The Generalised Predictive Control (GPC) method [12, 13] was applied. A recursive algorithm, which enables computation of predictions for arbitrary horizons, was designed.

The basic structure of the MPC is shown in Fig. 1. A model is used to predict the future process outputs y, based on the past and current values and on the proposed optimal future actions (manipulated variables) u. These actions are calculated by the optimizer taking into account the cost function (where the future tracking error is considered) as well as the constraints.

The paper is organized in the following way: problems of implementation of predictive control based on the minimization of the quadratic criterion (MPC QC)

* Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín, Nad Stráněmi 4511, 760 05 Zlín 5, Czech Republic, bobal@fai.utb.cz

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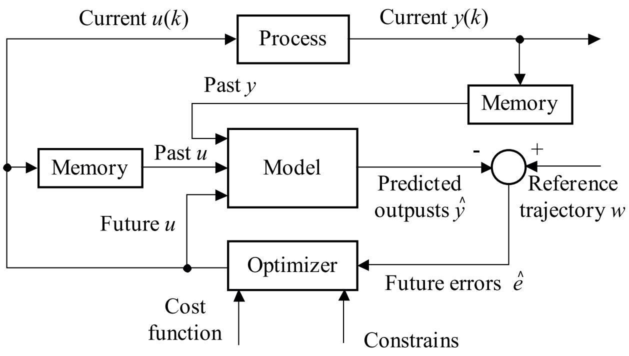

Fig. 1. Block diagram of basic structure of MPC

are described in Section 2 and on the minimisation of the absolute values criterion (MPC AC) in Section 3. The recursive identification for calculation of the parameter estimates is briefly introduced in Section 4. Description of the laboratory model Amira DR300 and is the content of Section 5. Section 6 presents an adaptive predictive control in the real-time conditions of the Amira DR300 servo system. Results obtained by application of quadratic cost function and absolute values cost function are compared in this section. Section 7 concludes the paper.

2 MPC BASED ON MINIMISATION OF QUADRATIC CRITERION

The standard cost function used in GPC contains quadratic terms of control error and control increments on a finite horizon into the future

$$J=\sum_{i=N_1}^{N_2}\big[\hat{y}(k+i)-w(k+i)\big]^2+\sum_{i=1}^{N_u}\big[\lambda(i)\Delta u(k+i-1)\big]^2 \tag{1}$$

where $\hat{y}(k+i)$ is the process output of i steps in the future predicted on the base of information available upon the time k, $w(k+i)$ is the sequence of the reference signal and $\Delta u(k+i-1)$ is the sequence of the future control increments that have to be calculated. Parameters N_1, N_2 and N_u are called minimum, maximum and control horizon. The parameter $\lambda(i)$ is a sequence which affects future behaviour of the controlled process, generally, it is chosen in the form of constants or exponential weights. The output of the model (predictor) is computed as the sum of the forced response $\boldsymbol{y}_n$ and the free response $\boldsymbol{y}_0$

$$\hat{\boldsymbol{y}}=\boldsymbol{y}_n+\boldsymbol{y}_0\,. \tag{2}$$

It is possible to compute the forced response as the multiplication of the matrix $\mathbf{H}$ (Jacobian Matrix of the model) and the vector of future control increments $\Delta\boldsymbol{u}$, which is generally a priori unknown

$$\boldsymbol{y}_n=\mathbf{H}\Delta\boldsymbol{u} \tag{3}$$

where

$$\mathbf{H}=\begin{bmatrix} h_1 & 0 & 0 & \dots & 0\\ h_2 & h_1 & 0 & \dots & 0\\ h_3 & h_2 & h_1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ h_{N_2} & h_{N_2-1} & h_{N_2-2} & \dots & h_{N_2-N_u+1}\end{bmatrix} \tag{4}$$

is matrix containing step responses.

It follows from equations (3) and (4) that the predictor in a vector form is given by

$$\hat{\boldsymbol{y}}=\mathbf{H}\Delta\boldsymbol{u}+\boldsymbol{y}_0\,. \tag{5}$$

Minimization of the cost function (1) now becomes a direct problem of linear algebra. The solution in an unconstrained case can be found by setting partial derivative of J with respect to $\Delta\boldsymbol{u}$ to zero and yields

$$\Delta\boldsymbol{u}=-(\mathbf{H}^\top\mathbf{H}+\lambda\mathbf{I})^{-1}\mathbf{H}^\top(\boldsymbol{y}_0-\boldsymbol{w})=-\mathbf{H}_H^{-1}\boldsymbol{g} \tag{6}$$

where $\mathbf{H}_H$ and $\boldsymbol{g}$ are the Hesse-Matrix and the gradient. Denoting the first row of the matrix $\big(\mathbf{H}^\top\mathbf{H}+\lambda\mathbf{I}\big)^{-1}\mathbf{H}^\top$ by $\boldsymbol{K}$, the actual control increment can be calculated as

$$\Delta u(k)=\boldsymbol{K}(\mathbf{w}-\boldsymbol{y}_0)\,. \tag{7}$$

2.1 Derivation of the prediktor for second order model

It resulted from identification experiments that the dynamical behaviour of the model DR300 can be described by second order model in individual set points. Actual parameters of the model depend on the set point. Let us consider a SISO process with the denominator and numerator polynomials in the form

$$\begin{aligned} A\big(z^{-1}\big)&=1+a_1z^{-1}+a_2z^{-2};\\ B\big(z^{-1}\big)&=b_1z^{-1}+b_2z^{-2}.\end{aligned} \tag{8}$$

Let us assume that $C\big(z^{-1}\big)=1$, the CARIMA description of the system is in the form

$$\Delta A\big(z^{-1}\big)y(k)=B\big(z^{-1}\big)\Delta u(k)+n_c(k)\,. \tag{9}$$

The non-measurable random component $n_c(k)$ is assumed to have zero mean value $\mathrm{E}[n_c(k)]=0$ and constant covariance (dispersion) $R=\mathrm{E}[n_c^2(k)]$.

The difference equation of the CARIMA model without the unknown term $n_c(k)$ can be expressed as

$$\begin{aligned} y(k)=(1-a_1)y(k-1)+(a_1-a_2)y(k-2)+a_2y(k-3)\\ +b_1\Delta u(k-1)+b_2\Delta u(k-2)\,. \end{aligned} \tag{10}$$

It was necessary to compute three step ahead predictions in straightforward way by establishing of lower predictions to higher predictions. The model order defines that computation of one step ahead prediction. It is based on three past values of the system output in case of a second order model. The three step ahead predictions are as follows

$$\begin{aligned}\hat{y}(k+1) &= (1-a_1)y(k) + (a_1-a_2)y(k-1) \\ &\quad + a_2 y(k-2) + b_1\Delta u(k) + b_2\Delta u(k-1)\,, \\ \hat{y}(k+2) &= (1-a_1)y(k+1) + (a_1-a_2)y(k) \\ &\quad + a_2 y(k-1) + b_1\Delta u(k+1) + b_2\Delta u(k)\,, \\ \hat{y}(k+3) &= (1-a_1)y(k+2) + (a_1-a_2)y(k+1) \\ &\quad + a_2 y(k) + b_1\Delta u(k+2) + b_2\Delta u(k+1)\,.\end{aligned} \tag{11}$$

The predictions after modification can be written in a matrix form

$$\begin{aligned}\begin{bmatrix}\hat{y}(k+1)\\ \hat{y}(k+2)\\ \hat{y}(k+3)\end{bmatrix} &= \begin{bmatrix} g_1 & 0\\ g_2 & g_1\\ g_3 & g_2\end{bmatrix}\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\end{bmatrix} \\ &\quad + \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14}\\ p_{21} & p_{22} & p_{23} & p_{24}\\ p_{31} & p_{32} & p_{33} & p_{34}\end{bmatrix}\begin{bmatrix} y(k)\\ y(k-1)\\ y(k-2)\\ \Delta u(k-1)\end{bmatrix} = \\ &\begin{bmatrix} b_1 & 0\\ b_1(1-a_1)+b_2 & b_1\\ (a_1-a_2)b_1+(1-a_1)^2 b_1 + (1-a_1)b_2 & b_1(1-a_1)+b_2\end{bmatrix}\times \\ &\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\end{bmatrix} + \begin{bmatrix} (1-a_1) & (a_1-a_2) & a_2 & b_2\\ (1-a_1)^2+(a_1-a_2) & (1-a_1)(a_1-a_2)+a_2 & a_2(1-a_1) & b_2(1-a_1)\\ (1-a_1)^3+2(1-a_1)(a_1-a_2)+a_2 & (1-a_1)^2(a_1-a_2)+a_2(1-a_1)+(a_1-a_2)^2 & a_2(1-a_1)^2+(a_1-a_2)a_2 & b_2(1-a_1)^2+(a_1-a_2)b_2\end{bmatrix}\times \\ &\begin{bmatrix} y(k)\\ y(k-1)\\ y(k-2)\\ \Delta u(k-1)\end{bmatrix}\end{aligned} \tag{12}$$

It is possible to divide computation of the predictions to recursion of the free response and recursion of the matrix of the dynamics. Based on the three previous predictions it is repeatedly computed the next row of the free response matrix in the following way

$$\begin{aligned} p_{41} &= (1-a_1)p_{31} + (a_1-a_2)p_{21} + a_2 p_{11}\,, \\ p_{42} &= (1-a_1)p_{32} + (a_1-a_2)p_{22} + a_2 p_{12}\,, \\ p_{43} &= (1-a_1)p_{33} + (a_1-a_2)p_{23} + a_2 p_{13}\,, \\ p_{44} &= (1-a_1)p_{34} + (a_1-a_2)p_{24} + a_2 p_{14}\,. \end{aligned} \tag{13}$$

The first row of the matrix is omitted in the next step and further prediction is computed based on the three last rows including the one computed in the previous step. This procedure is cyclically repeated. It is possible to compute an arbitrary number of rows of the matrix.

The recursion of the dynamics matrix is similar. The next element of the first column is repeatedly computed in the same way as in the previous case and the remaining columns are shifted to form a lower triangular matrix in the way which is obvious from the equation (12). This procedure is performed repeatedly until the prediction horizon is achieved. If the control horizon is lower than the prediction horizon a number of columns in the matrix is reduced. Computation of the new element is performed as follows

$$g_4 = (1-a_1)g_3 + (a_1-a_2)g_2 + a_2 g_1\,. \tag{14}$$

2.2 FORMULATION OF OPTIMAL CONTROL WITH CONSTRANTS

In case of the Amira DR300 laboratory model, the actuator has a limited range of action. The voltage applied to the motor must be within fixed limits. As it was mentioned in the Section 1, MPC can consider constrained input and output signals in the process of the controller design. General formulation of predictive control with constraints is then as follows

$$\min_{\Delta \boldsymbol{u}} 2\boldsymbol{g}^\top \Delta\boldsymbol{u} + \Delta\boldsymbol{u}^\top \mathbf{H}_H \Delta\boldsymbol{u} \tag{15}$$

owing to

$$\mathbf{A}\Delta\boldsymbol{u} \le \boldsymbol{b}\,. \tag{16}$$

The inequality (16) expresses the constraints in a compact form. In our case of the constrained input signals particular matrices can be expressed as

$$\mathbf{A} = \begin{bmatrix}\mathbf{T}\\ -\mathbf{T}\end{bmatrix}; \quad \boldsymbol{b} = \begin{bmatrix}\boldsymbol{1}u_{\min} - \boldsymbol{1}u(k-1)\\ -\boldsymbol{1}u_{\max} + \boldsymbol{1}u(k-1)\end{bmatrix} \tag{17}$$

where $\mathbf{T}$ is a lower triangular matrix whose non-zero elements are ones and $\boldsymbol{1}$ is vector of ones.

Forms of the matrices for an arbitrary control horizon were computed and can be expressed as follows

$$\begin{bmatrix} -1 & 0 & \dots & 0\\ \vdots & \vdots & \dots & \vdots\\ -1 & -1 & \dots & -1\\ 1 & 0 & \dots & 0\\ \vdots & \vdots & \dots & \vdots\\ 1 & 1 & \dots & 1\end{bmatrix}\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\\ \vdots\\ \Delta u(k+N_u-1)\end{bmatrix} \le \begin{matrix} -\begin{bmatrix}1\\1\\ \vdots\\ 1\end{bmatrix}u_{\min} + \begin{bmatrix}1\\1\\ \vdots\\ 1\end{bmatrix}u(k-1)\\ \begin{bmatrix}1\\1\\ \vdots\\ 1\end{bmatrix}u_{\max} - \begin{bmatrix}1\\1\\ \vdots\\ 1\end{bmatrix}u(k-1)\end{matrix} \tag{18}$$

The control sequence is computed from expression (15), equations (17) and inequalities (16), (18). The optimization problem is solved numerically by quadratic programming in each sampling period. The first element of the resulting vector is then applied as the increment of the manipulated variable.

3 MPC BASED ON MINIMISATION OF ABSOLUTE VALUES CRITERION

Model predictive control can be also formulated for the criterion based absolute values of predicted control error, future control values and future control values differences

$$J = \sum_{i=N_1}^{N_2} \left|\hat{y}(k+i) - w(k+i)\right| + \sum_{i=1}^{N_u} \lambda_1(i)\left|u(k+i-1)\right| + \sum_{i=1}^{N_u} \lambda_2(i)\left|\Delta u(k+i-1)\right| \quad (19)$$

where k is the current control step, $\hat{y}(k+i)$ is the process output in step $k+i$ predicted using information available upon the time k, $w(k+i)$ is the reference signal in step $k+i$ and $u(k+i-1)$ are the future control values that have to be calculated. Parameters N_1, N_2 and N_u are minimum, maximum and control horizon. The parameters $\lambda_1(i)$ and $\lambda_2(i)$ are a sequences which affects future behaviour of the controlled process. Generally, they are chosen in the form of constants or exponential weights. The criterion can be also considered as sum of 1-norm of predicted control errors and weighted 1-norm of future control actions and its differences. Contrary to MPC based on quadratic criterion, a direct penalisation of control signal can be incorporated even for proportional controlled systems while reaching zero steady control error.

The derivation of the control law is generally similar to the derivation presented in the chapter 2. The output of the model (predictor) is computed as the sum of the forced response $\boldsymbol{y}_n$ and the free response $\boldsymbol{y}_0$

$$\hat{\boldsymbol{y}} = \boldsymbol{y}_n + \boldsymbol{y}_0 \,. \quad (20)$$

It is possible to compute the forced response as the multiplication of the matrix $\mathbf{G}$ and the vector of future control values $\boldsymbol{u}$, which is generally a priori unknown

$$\boldsymbol{y}_n = \mathbf{G}\boldsymbol{u} \quad (21)$$

where

$$\mathbf{G} = \begin{bmatrix} g_1 & 0 & 0 & \dots & 0 \\ g_2 & g_1 & 0 & \dots & 0 \\ g_3 & g_2 & g_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{N_2} & g_{N_2-1} & g_{N_2-2} & \dots & \sum_{i=1}^{N_2-N_u+1} g_i \end{bmatrix} \quad (22)$$

is matrix containing impulse responses.

It follows from equations (21) and (22) that the predictor in a vector form is given by

$$\hat{\boldsymbol{y}} = \mathbf{G}\boldsymbol{u} + \boldsymbol{y}_0 \,. \quad (23)$$

The problem of minimizing the criterion (19) can be reformulated as linear programming problem:

$$\min_{\boldsymbol{t}} \boldsymbol{g}_1^\top \boldsymbol{t} \quad (24)$$

where

$$\boldsymbol{g}_1^\top = \begin{bmatrix} \boldsymbol{1}_{1\times N_2-N_1+1} & \lambda_1(1) \dots \lambda_1(N_2) & \lambda_2(1) \dots \lambda_2(N_2) & \boldsymbol{0}_{1\times N_u} \end{bmatrix} \quad (25)$$

and

$$\boldsymbol{t}^\top = \begin{bmatrix} \boldsymbol{r}^\top_{(N_2-N_1+1)\times 1} & \boldsymbol{s}^\top_{1N_u\times 1} & \boldsymbol{s}^\top_{2N_u\times 1} & \boldsymbol{u}^\top_{N_u\times 1} \end{bmatrix}. \quad (26)$$

Vectors $\boldsymbol{t}$, $\boldsymbol{r}$, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are auxiliary vectors. Their subscripts in (26) represent the sizes. Minimization is owing to constraints

$$-\boldsymbol{r} \le \mathbf{G}\boldsymbol{u} + \boldsymbol{y}_0 - \boldsymbol{w} \le \boldsymbol{r}\,, \quad (27)$$

$$-\boldsymbol{s}_1 \le \boldsymbol{u} \le \boldsymbol{s}_1\,, \quad (28)$$

$$-\boldsymbol{s}_2 \le \Delta\boldsymbol{u} \le \boldsymbol{s}_2\,, \quad (29)$$

$$\boldsymbol{0} \le \boldsymbol{r}\,, \quad \boldsymbol{0} \le \boldsymbol{s}_1\,, \quad \boldsymbol{0} \le \boldsymbol{s}_2\,. \quad (30)$$

Inequalities (27), (28) and (29) correspond to the first, second and third sum in the criterion (19) respectively. Vector of control signal differences ($\Delta\boldsymbol{u}$) can be expressed by vector $\boldsymbol{u}$ and previous value of control signal $u(k-1)$

$$\Delta\boldsymbol{u} = \begin{bmatrix} u(k) - u(k-1) \\ u(k+1) - u(k) \\ \vdots \\ u(k+N_u-1) - u(k+N_u-2) \end{bmatrix} = \begin{bmatrix} -1 & 1 & & & 0 \\ 0 & -1 & 1 & & \\ \vdots & & \ddots & \ddots & \\ 0 & 0 & & -1 & 1 \end{bmatrix} \begin{bmatrix} u(k-1) \\ u(k) \\ \vdots \\ u(k+N_u-1) \end{bmatrix} = \boldsymbol{r}_1 u(k-1) + \boldsymbol{R}_2\boldsymbol{u}\,. \quad (31)$$

where $\boldsymbol{r}_1$ is the first column of the matrix in equation (31) and $\boldsymbol{R}_2$ is a submatrix of the matrix in equation (31) without the first column. Handling of the linear constraints is similar to the handling of quadratic programming constraints in chapter 2.2. The difference consists in using control values instead of their differences

$$\mathbf{A}_1\boldsymbol{u} \le \boldsymbol{b}_1\,. \quad (32)$$

The inequality (32) expresses the constraints of manipulated variable in a compact form. In our case of the constrained input signal, the matrix $\mathbf{A}_1$ and vector $\boldsymbol{b}_1$ can be expressed as

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{I} \\ -\mathbf{I} \end{bmatrix}; \quad \boldsymbol{b}_1 = \begin{bmatrix} \boldsymbol{1}u_{\min} \\ -\boldsymbol{1}u_{\max} \end{bmatrix} \quad (33)$$

where $\mathbf{I}$ is an identity matrix and $\boldsymbol{1}$ is the unity vector. Contingent constraints of the control signal differences are handled in a similar way using (31).

As $\boldsymbol{t}$ is a vector used in minimization (24), all constraints of linear programming stated by inequalities are formulated as follows

$$\begin{bmatrix} -\mathbf{I} & \mathbf{0} & \mathbf{0} & -\mathbf{G} \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{G} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & -\mathbf{r}_2 \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{r}_2 \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{u} \end{bmatrix} \leq \begin{bmatrix} \boldsymbol{y}_0 - \boldsymbol{w} \\ -\boldsymbol{y}_0 + \boldsymbol{w} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{r}_1 u(k-1) \\ -\mathbf{r}_1 u(k-1) \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{b}_1 \end{bmatrix}. \tag{34}$$

The element $u(k)$ obtained by numerical solution of linear programming is then applied as the manipulated variable.

4 RECURSIVE IDENTIFICATION

The regression ARX model in the following form is used in the identification part of the designed controller

$$y(k) = \hat{\boldsymbol{\Theta}}^\top(k)\boldsymbol{\Phi}(k) + n(k) \tag{35}$$

where

$$\hat{\boldsymbol{\Theta}}^\top(k) = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \hat{b}_1 & \hat{b}_2 \end{bmatrix} \tag{36}$$

is the vector of the parameter estimates and

$$\boldsymbol{\Phi}^\top(k) = [\,-y(k-1) \;\; -y(k-2) \;\; u(k-1) \;\; u(k-2)\,] \tag{37}$$

is the regression vector. The parameter estimates (36) are computed using the recursive least squares method. Numerical stability is improved by means of the LD decomposition and the adaptation is supported by the directional forgetting [19–21]. The initial parameter estimates were set to the values obtained at the end of the previous experiment.

5 DESCRIPTION OF LABORATORY MODEL DR 300

The proposed self-tuning (MPC) algorithms were tested using a real-time laboratory model DR300 (Speed Control with Variable Load) by the Amira Company, Duisburg, Germany (see Fig. 2).

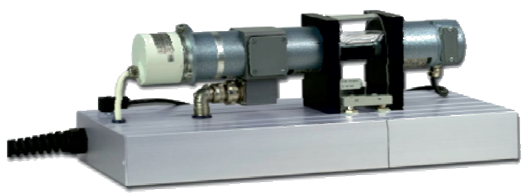

Fig. 2. Laboratory model Amira DR300

A block scheme of this system is shown in Fig. 3. The plant is represented by a permanently exited DC-motor (M1) whose input signal (armature current) is provided by a controller. The sensors for the output signal (rotation) are a tachogenerator (T) and an incremental encoder (I). The free end of the motor shaft is fixedly coupled (K) to the shaft of a second, identical motor (M2). This motor can be used as a generator. The rotation speed of the motor M1 is driven by voltage u. The aim of the control process is to control the rotation speed of the motor M1 shaft ω (rpm) (rotations per minute) with the control voltage u (V).

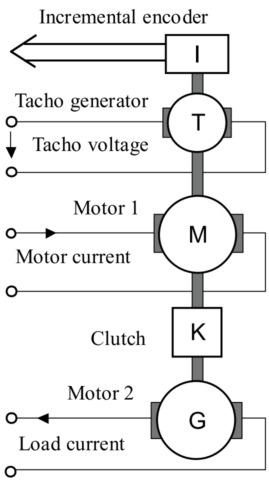

Fig. 3. Simplified scheme of Amira DR300 laboratory servo system

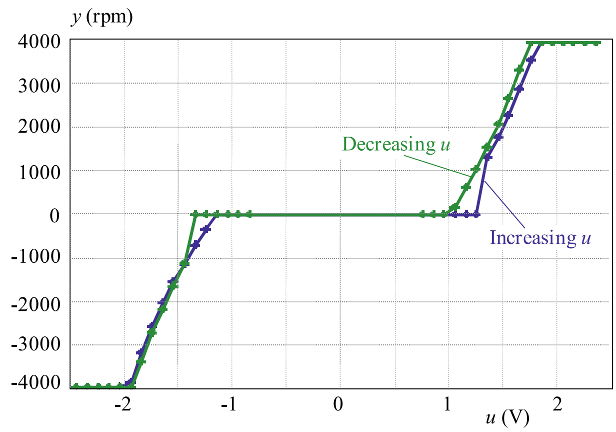

Fig. 4. Static characteristics of DR300 servo system

From the control point of view, the Amira DR300 is a non-linear system. Some characteristics of the nonlinearity (gain with dead zone and hysteresis) can be observed from the static characteristics shown in Fig. 4.

It is obvious that friction plays a big role in this control problem. The control value in the approximate range from -1 V to $+1$ V does not cause a rotation of the shaft. Difference between static and kinetic friction can be observed for control signal u around $+1$ V (and -1 V). If

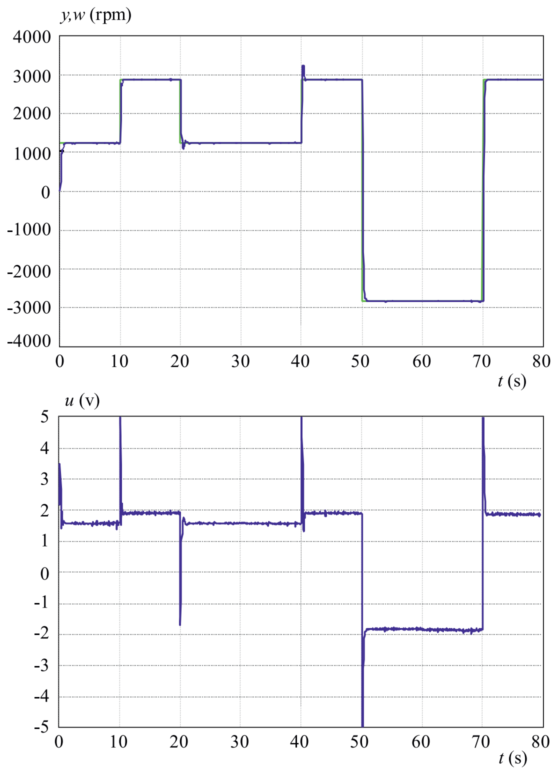

Fig. 5. Control of DR300 using STMPC QC (unconstrained case)

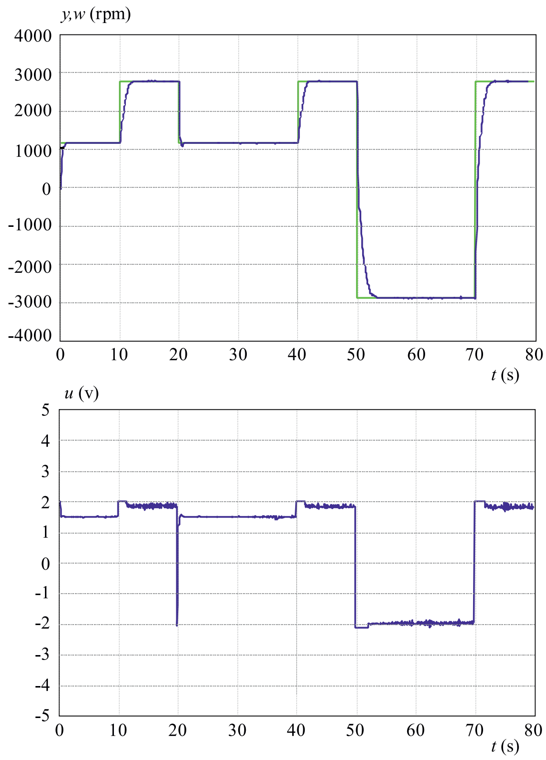

Fig. 6. Control of DR300 using STMPC QC (constrained case)

the shaft is not rotating, control signal must be increased up to 1.4 V to start shaft rotation. On the other hand, if the shaft is rotating, decreasing control signal down to 1.1 V still causes small shaft rotation. In addition, a steady control signal below -2 V or above $+2$ V leads to saturation. Moreover, the static characteristics in the remaining ranges from -2 V to -1 V and from $+1$ V to $+2$ V are not strictly linear. Another problem consists in changes of overall gain of the system. These changes can be caused by external conditions of the real-time control process (humidity, temperature, *etc*).

6 REAL–TIME IMPLEMENTATION OF ST PREDICTIVE CONTROLLERS

6.1 ST predictive controller based on minimization of quadratic criterion

The aim of this chapter is implementation of the self-tuning model predictive controller based on minimization of quadratic criterion (STMPC QC) for control of the objective laboratory equipment DR300 servo-system. Courses of the reference signal contain step changes in both directions. This is one of the most unfavourable situations which can occur in a closed loop control system because the operational range also changes within the steps. This is one of the reasons for application of self-tuning controllers.

An approximate sampling period was found based on measured step responses so that ten samples cover important part of the step response. The best sampling period $T_0 = 0.05$ s was then tuned according to experiments.

The tuning parameters — the prediction and control horizons and the weighting coefficient λ — were tuned experimentally. There is a lack of clear theory relating to the closed loop behaviour to design parameters. The prediction horizon, which should cover the important part of the step response, was set to $N_2 = 15$. The control horizon was also set to $N_u = 15$. The coefficient λ was taken as equal to 50.

Both constrained and unconstrained cases were considered. Control results when constraints of the manipulated variable were not considered are presented in Fig. 5

In the subsequent experiment the manipulated variable was constrained within lower and upper limits and the algorithm considering the constraints was applied. The results are shown in Fig. 6.

6.2 ST predictive controller based on minimization of absolute values criterion

Application of the self-tuning model predictive controller based on absolute values criterion (STMPC AVC) to the DR300 system is presented in this chapter. Course of the reference signal and the sampling period are the same as in the previous paragraph (STMPC QC).

Control courses for the settings of $N_1 = 1$, $N_2 = 10$ and $N_u = 5$ are presented in the Fig. 7. All coeffi-

Table 1. Control quality criteria

	$S_{e2}(10^4\text{rpm}^2)$	$S_{du2}(10^{-3}\text{V}^2)$	$S_{ea}(\text{rpm})$	$S_{dua}(10^{-2}\text{V})$
STMPC QC (constrained case)	8.10	3.50	66.17	1.59
STMPC QC (unconstrained case)	11.94	1.89	92.15	1.15
STMPC AVC	2.32	127.78	29.09	15.67

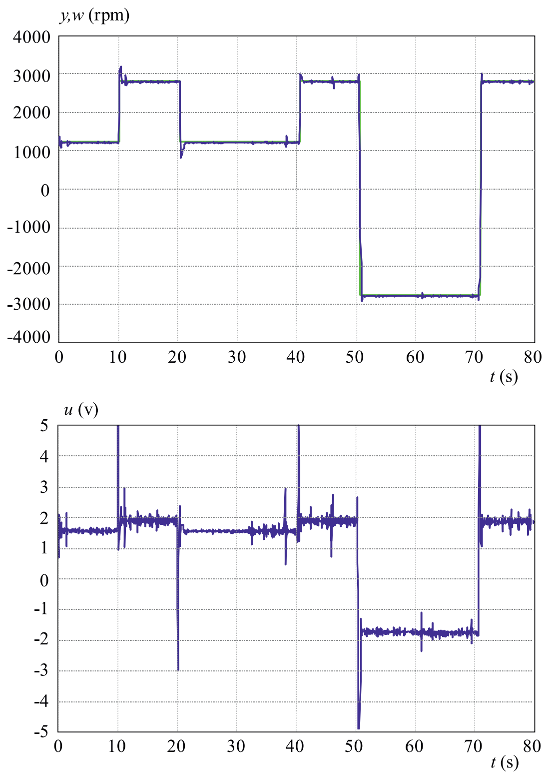

Fig. 7. Control of DR300 using STMPC AVC

cients λ_1 and λ_2 were equal to 0.2 in this case. The controller was selected from the STuMPCoL (Self-Tuning Model Predictive Controllers Library) designed for MATLAB/Simulink environment [22].

Utilization of absolute values criterion leads to faster response of the controller to the step changes of the reference signal. On the other hand, small changes of the control error caused by noise have greater influence to the control signal when comparing control courses with the control courses obtained by MPC based on quadratic criterion.

6.3 Numeric comparison of results

Control courses obtained by STMPC QC and STMPC AVC can be compared either visually by observing control courses presented in Figs. 5–7 or numerically using summing criteria of control quality. Four criteria based on control error and control signal differences were used

$$\begin{aligned} S_{e2} &= \frac{1}{b-a+1}\sum_{k=a}^{b}[e(k)]^2, \\ S_{ea} &= \frac{1}{b-a+1}\sum_{k=a}^{b}|e(k)|, \\ S_{du2} &= \frac{1}{b-a+1}\sum_{k=a}^{b}[\Delta u(k)]^2, \\ S_{dua} &= \frac{1}{b-a+1}\sum_{k=a}^{b}|\Delta u(k)|. \end{aligned} \tag{38}$$

Resulting values of the criteria are presented in Tab. 1. Values of limits a and b were chosen to cover whole course.

7 CONCLUSIONS

The contribution presented self-tuning model-based predictive control applied to a highly nonlinear proportional servo system. A linear model with constant coefficients used in pure model predictive control cannot describe the control system in all its modes. Therefore, an on-line identification was incorporated into the controller to obtain self-tuning capabilities.

MPC based on quadratic and absolute values criterion were derived and tested. The Amira DR300 servo system was used for verification of proposed controllers. The experiments confirmed that both STMPC QC and STMPC AVC approaches are able to cope with the given control problem. The courses obtained by the controller based on quadratic criterion are smoother then the courses obtained by controller based on absolute value criterion. A reason for this behaviour consists in greater influence of noise to the control signal when using absolute values criterion.

Industrial applications of MPC are usually based on optimization of economic factors while preserving technological constraints. This task can be usually represented by either quadratic or absolute values criterion. Thus, choice between quadratic criterion and absolute values criterion should depend on individual application of the MPC.

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Vladimír Bobál (prof, Ing, CSc) graduated from the Faculty of Mechanical Engineering, Brno University of Technology, Czech Republic in 1966. He received his CSc. degree in Technical Cybernetics at Institute of Technical Cybernetics, Slovak Academy of Sciences in Bratislava, Slovak Republic. From 1969 he has been working as a senior lecturer, associate Professor and now as Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín, Czech Republic. His research interests are adaptive control systems, predictive control and system identification. He is author or co-author of 2 scientific books and over 270 contributions in scientific journals and conference proceedings.

Petr Chalupa (Ing, PhD) was born in Zlín, Czech Republic in 1976. He graduated from Brno University of Technology in 1999. He obtained his PhD in Technical Cybernetics at Tomas Bata University in Zlin in 2003. He works as a researcher at Center of Applied Cybernetics at Tomas Bata University in Zlin. His research interests are adaptive and predictive control of real-time systems. He is author or co-author of 1 scientific books and of over 70 contributions in scientific journals and conference proceedings.

Marek Kubalčík (doc, Ing, PhD) was born in Zlín, Czech Republic. He graduated in 1993 from the Brno University of Technology in Automation and Process Control. He received his Ph.D. degree in Technical Cybernetics at Brno University of Technology in 2000. From 1993 to 2007 he worked as a senior lecturer at the Faculty of Technology, Brno University of Technology, Czech Republic. From 2007 he has been working as an associate Professor at the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín, Czech Republic. Current work covers following areas: control of multivariable systems, selftuning controllers, predictive control. He is an author or co-author of over 50 publications in journals and conference proceedings.

Petr Dostál (prof, Ing, CSc) was born in Kněždub, Czech Republic in 1945. He graduated from the VŠCHT Pardubice in Chemical Engineering and Process Control in 1968. He received his CSc degree in Technical Cybernetics at VŠCHT Pardubice in 1979. From 1971 to 1996 he worked as a senior lecturer and an associate Professor at the Faculty of Chemical Technology STU Bratislava, Slovak Republic. From 1997 he has been working as associate Professor and Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín, Czech Republic. Current work covers two main areas: algebraic control theory with focus on polynomial methods and adaptive control of nonlinear technological processes. He is author or co-author of 2 scientific books and over 240 publications in journals and conference proceedings.